

Sheath instabilities in Hall plasmas devices

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New class instabilities is identified in Hall plasmas in configurations with open magnetic field lines. It is shown that sheath resistivity results in a robust instability driven by the equilibrium electric field. It is conjectured that these instabilities play a crucial role in anomalous transport in Hall plasmas devices.

1. Introduction. Plasmas in contact with material surface form a narrow region of noncompensated positive charge, or sheath, which occurs as a result of fast losses of negative charge due to the high mobility of electrons. This basic phenomena is of fundamental importance for various plasma physics applications. The width of the sheath region, typically of the order of the electron Debye length, $\lambda_D = (T_e/4\pi e^2 n_0)^{1/2}$, is normally many orders of magnitude smaller than the characteristic length scale of the bulk plasma region. Despite such large length scale separation sheath may critically affect the properties of bulk plasma, e.g. anomalous (turbulent) transport in magnetically confined plasmas [1, 2].

While the turbulent transport in plasmas has been studied for many years and there have been significant advances in understanding of dynamics of drift wave turbulence in magnetized plasmas, the complete understanding remains elusive. One of the less studied field is the anomalous mobility of electrons in Hall plasmas[3]. Hall plasmas, with a characteristic geometrical length scale between the ion and electron Larmor radii, $\rho_e \ll L \ll \rho_i$, and intermediate range of the frequencies between ion and electron cyclotron frequencies, $\omega_{ci} \ll \omega \ll \omega_{ce}$, represent a special, but common and important case of magnetically controlled plasma. Hall plasmas conditions are typical for many laboratory and space plasmas, e.g. for some regions in Earth ionosphere and magnetosphere.

Hall plasma conditions are typical for many technological applications. Common feature of these devices is presence of stationary externally applied electric field \mathbf{E}_0 , which is perpendicular to the equilibrium magnetic field \mathbf{B}_0 , and produce stationary $\mathbf{E}_0 \times \mathbf{B}_0$ drift velocity. The ions, due their large Larmor radius, are unmagnetized and accelerated in the \mathbf{E}_0 direction, while the electron collisions lead to a finite current along the \mathbf{E}_0 . As a result, quasineutral plasma is accelerated along the \mathbf{E}_0 . Such plasma accelerators, typically in coaxial geometry with radial magnetic field \mathbf{B}_0 , axial \mathbf{E}_0 , and azimuthal drift $\mathbf{E}_0 \times \mathbf{B}_0$, also used as technological plasma sources, have recently became a subject of growing interest due their applications in electric space propulsion, so called Hall plasma thrusters.

Despite many years of successful operation in space [4], the physics of the Hall thrusters is not completely understood. Ongoing expansion of applications of Hall thrusters for various space missions of growing complexities and associated new requirements dictate the need for better understanding of physical processes in these devices[5]. One of the critical problems, is the nature of the electron current across the magnetic field. Electrons are nearly collisionless in typical applications, and collisional values of the current are too low to explain the experimental data. It has long been known that Hall thruster plasmas are typically in a turbulent state exhibiting a multitude of instabilities and fluctuations in the wide range of frequencies[6]. Plasma turbulence was suggested to be the reason for anomalous electron mobility, however the exact nature of responsible instabilities remains unclear. At the same time, an alternative mechanism of electron conductivity created by electron collisions with the walls, so called near-wall conductivity, was proposed[4]. However, the reflection of electrons from the ideal sheath would be reversible, so the stochasticisation mechanism of near-wall conductivity is unclear. In the original work[4], it was suggested that sheath fluctuations would create the required stochastic motion of electron, though the reasons for such fluctuations are unknown. Experimental data and numerical modeling suggest[7] that a combination of both turbulent mobility and near-wall conductivity is required to satisfactory describe experimental behavior.

In this paper, we propose a novel instability mechanism based on coupling of sound waves in bulk plasma with sheath fluctuations. Sheath dynamics, included as a boundary condition for main plasma, provides the positive feedback rendering waves unstable. Such mechanism affects both turbulent mobility of bulk electrons and near-wall conductivity of electrons reflected by fluctuating sheath. Recent numerical simulations have also suggested the presence of sheath fluctuations[8, 9]. There has been a number of previous studies which concentrated on instabilities in crossed $\mathbf{E}_0 \times \mathbf{B}_0$ fields Hall plasmas, which were caused by the equilibrium electron flow in combination with density and magnetic field gradients, resistive effects, and electron cyclotron effects. These instabilities may

also be affected by sheath impedance effects and eb operative in addition to the instability of basic sound waves which is studied in this paper.

It was noted earlier that boundary conditions at the sheath amounts to a specific dissipation for bulk plasma, the so called sheath resistivity[15]. Sheath resistivity was shown to lead to some instabilities of magnetically confined plasmas for fusion applications[2, 16] but was not studied in Hall plasmas conditions. Original concept of sheath resistivity was developed for conducting walls. For such a wall, the electron and ion currents into the wall should not be balanced locally. Therefore, local perturbations of plasma density and potential lead to different ion and electron currents, thus producing a finite current into the sheath. For the dielectric wall, the current on the material surface must be zero. However, the total sheath impedance has a finite and large reactive(capacitive) part, so it is able to maintain a finite oscillating current on the quasineutral plasma side even when the current into the wall is zero. Recognition of this fact allows us to describe the sheath effects as a boundary condition for bulk plasma oscillations. It turns out that in this formulation, the resistive part of the total sheath impedance leads to a robust instability in $\mathbf{E}_0 \times \mathbf{B}_0$ Hall plasmas.

Principally, the stationary $\mathbf{E}_0 \times \mathbf{B}_0$ flow of magnetized electrons of the velocity is a powerful source of free energy in Hall plasma devices. The electron perturbations with phase velocities lower than the equilibrium flow velocity $\mathbf{v}_0 = c\mathbf{E}_0 \times \mathbf{B}_0/B_0^2$ will have negative energy: the total energy of the perturbed system is lower than the energy of the unperturbed state with equilibrium flow. The negative energy perturbations are prone to dissipation instabilities. Dissipation takes the energy away from the negative energy perturbation thus inducing the further growth of its amplitude. We illustrate this mechanism with a simple example of the ion sound instability in Hall plasma with equilibrium electron flow in Section II. The growth of the negative energy modes induced by sheath dissipation is the exactly the instability mechanism for fluctuations in Hall plasma plasmas studied in Section V. This instability affects both turbulent mobility and near wall conductivity.

2. *Destabilized ion sound waves in Hall plasmas.* Ions are unmagnetized and described by the equation

$$-i\omega m_i \mathbf{V}_i = -e\nabla\tilde{\phi}, \quad (1)$$

We consider the dynamics along the magnetic field and in perpendicular direction separately, so that

$$\nabla = \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z}, \quad (2)$$

where z is the direction along the magnetic field, $\hat{\mathbf{z}}$ is a unit vector along the equilibrium magnetic field, and $\hat{\mathbf{y}}$ is the unit vector in the azimuthal direction. Then ion continuity equation takes the form

$$-i\omega \tilde{n}_i - i \frac{en_0}{\omega m_i} \frac{\partial^2}{\partial y^2} \tilde{\phi} + \frac{1}{e} \frac{\partial}{\partial z} \tilde{J}_{\parallel i} = 0. \quad (3)$$

The electrons are magnetized and, in the lowest order, the perpendicular electron velocity is given by the

$$\mathbf{V}_{\perp e} = \mathbf{V}_{\perp 0} + \frac{c}{B_0} \mathbf{b} \times \nabla \tilde{\phi}, \quad (4)$$

$\mathbf{V}_{\perp 0} = c\mathbf{E}_0 \times \mathbf{b}/B_0^2 = -cE_{0x}/B_0 \hat{\mathbf{y}}$ is the equilibrium electron drift due to the axial electric field E_{0x} .

The electrons have large mobility along the magnetic field and have Boltzmann distribution of density

$$\tilde{n}_e = \frac{e\tilde{\phi}}{T_e} n_0. \quad (5)$$

It is valid for $\omega < k_z v_{Te}$ and is found from the electron momentum balance along the magnetic field

$$en \frac{\partial \tilde{\phi}}{\partial z} - T_e \frac{\partial \tilde{n}_e}{\partial z} = 0, \quad (6)$$

electrons are assumed to be isothermal $T_e = \text{const.}$ In the local approximation, Boltzmann electrons (5) and perturbed ion density from (3) result in the dispersion relation for ion sound waves

$$\omega^2 = (k_z^2 + k_y^2) c_s^2 \quad (7)$$

Note that the equilibrium electron drift $\mathbf{V}_{\perp 0}$ does not result in the Doppler frequency shift because ions are not magnetized here. In this derivation, we do not utilize the information about the electron current along the magnetic field, though obviously such a current is finite. It can be found from the electron continuity equation

$$-i(\omega - \omega_0) \tilde{n}_e - \frac{1}{e} \frac{\partial}{\partial z} \tilde{J}_{\parallel e} = 0. \quad (8)$$

where $\omega_0 = \mathbf{k} \cdot \mathbf{V}_{\perp 0}$. Using quasineutrality, the ion and electron continuity equations can be cast into the form that explicitly shows a contribution of the current along the magnetic field

$$-i\omega_0 \tilde{n}_e - i \frac{en_0}{\omega m_i} \frac{\partial^2}{\partial y^2} \tilde{\phi} + \frac{1}{e} \frac{\partial}{\partial z} \tilde{J}_{\parallel} = 0, \quad (9)$$

where $\tilde{J}_{\parallel} = \tilde{J}_{\parallel i} + \tilde{J}_{\parallel e}$. Together with (1), (5), and (8), this equation, of course, will result in (7).

The instability mechanism of the discussed modes in essential way relies on the stationary electron flow due to the equilibrium electric field \mathbf{E}_0 and presence of dissipation. To highlight the physical mechanism of the instability we consider a simple model of the ion sound waves in Hall plasma in presence of electron collisions. Taking into account the electron collisions, the parallel momentum balance becomes

$$k_z \left(\tilde{\phi} - \frac{T_e}{e} \tilde{n}_e \right) - \frac{m_e \nu_e}{e} (\tilde{v}_{ze} - \tilde{v}_{zi}) = 0. \quad (10)$$

Using this expression and the electron continuity one has for the density perturbations

$$\frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\phi}}{T_e} \frac{1}{1 - i\nu_e (\omega - \omega_0) / k_z^2 v_{Te}^2}. \quad (11)$$

Together with ion equation and quasineutrality it results in the dispersion relation for unstable ion sound waves

$$\omega^2 = k^2 c_s^2 - i \frac{k^2 c_s^2}{k_z^2 v_{Te}^2} (\omega - \omega_0). \quad (12)$$

This instability can be interpreted as the negative energy sound wave, $\omega < \omega_0$, destabilized by dissipation. For weakly collisional plasmas, when $\lambda_e > L\sqrt{m_e/m_i}$, the bulk dissipation due to interquartile collisions, becomes less important compared to the sheath dissipation [15], where λ_e is the electron mean free path, and L is the characteristic length scale of the plasma slab (in the direction perpendicular to the sheath). In Hall thruster plasmas, sheath dissipation induces unstable global and small scale modes, which are considered next.

4. *Boundary conditions and sheath impedance.* The sheath effects are considered by including the perturbed current along the magnetic field into the sheath. For stationary state, we assume the standard sheath model with Bohm ion current

$$J_{i0} = enc_s, \quad (13)$$

and the electron current

$$J_{e0} = -\frac{1}{2\sqrt{\pi}} en v_{Te} \exp(-e\phi_0/T_e). \quad (14)$$

The ion sound velocity is defined as $c_s^2 = T_e/m_i$. In stationary conditions the total current into the sheath is zero: $J_{0e} + J_{0i} = 0$.

Potential $\tilde{\phi}$, density \tilde{n} , and temperature fluctuations \tilde{T}_e at the sheath boundary produce the perturbations of the electron and ion current into the sheath

$$\tilde{J}_i = \frac{\tilde{n}_i}{n_0} J_{0i} + \frac{1}{2} \frac{\tilde{T}_e}{T_{e0}} J_{0i}, \quad (15)$$

$$\tilde{J}_e = \frac{\tilde{n}_e}{n_0} J_{0e} + \frac{1}{2} \frac{\tilde{T}_e}{T_{e0}} J_{0e} - \left(\frac{e\tilde{\phi}}{T_e} - \Lambda \frac{\tilde{T}_e}{T_{e0}} \right) J_{0e}, \quad (16)$$

where

$$\Lambda = \ln \left(\sqrt{\frac{m_i}{2\pi m_e}} \right). \quad (17)$$

In this model we do not consider the processes inside the sheath layer and do not specify how the finite current into the sheath is accommodated. The actual dynamics of this current will depend on the surface material and conditions, e.g. for the metal surface this current can be admitted into the wall, while for the dielectric the finite current into the sheath will lead to perturbation and rearrangement of the spatial charge distribution in the sheath layer and on the surface. The characteristic frequency of sheath oscillations is of the order of the ion plasma frequency ω_{pi} . Therefore, as long as the frequency of the perturbations is lower than ω_{pi} and the amplitude is small,

$$e\tilde{\phi}/T_e < \omega/\omega_{pi}, \quad (18)$$

the sheath perturbation will retain its standard structure (13) and (14), and the current at the sheath boundary can be determined as a small perturbation of (13) and (14). This condition alternatively can be viewed as the condition that the characteristic charge admitted into the sheath as a result of fluctuations \tilde{J}/ω is small compared to the charge of the stationary sheath Q ,

$$\tilde{J}/\omega < Q. \quad (19)$$

Then, the sheath will only experience a small perturbation and the current at the sheath boundary can be calculated from (13) and (14). The charge is the sheath layer $Q \simeq \rho_e d$, where the charge density ρ_e is estimated as $\rho_e \simeq \phi_0/(4\pi d^2)$, where the sheath thickness is of the order of the Debye length, $d \simeq \lambda_{De}$. Assuming that $\tilde{J} \simeq J_{0e}\tilde{\phi}/T_e$, one obtains the condition

$$\frac{e\tilde{\phi}}{T_e} < \frac{e\phi_0}{T_e} \frac{\omega \lambda_{De}}{c_s}, \quad (20)$$

which is equivalent to (18).

5. *Global sheath induced modes.* Presence of the sheath at the plasma boundary imposed additional constraints on plasma dynamics via the conditions (15) and (16). As a result of these constraints, the ion sound dynamics is modified and new modes appear. In general, these modes have the eigenmode structure that depends both on the z -coordinate and the perpendicular direction y . It is instructive first to consider the modes which have no structure along the magnetic field. Such modes can be derived within simple model equations averaged along the magnetic field lines. Since these modes have no profile along the magnetic field, $\partial/\partial z = 0$, we will call them the global modes.

Consider the magnetic field line tube between $z = -L$ and $z = L$ and introduce the averaged potential and density

$$\bar{n} \equiv \frac{1}{2L} \int_{-L}^L \tilde{n} dl, \quad (21)$$

$$\bar{\phi} \equiv \frac{1}{2L} \int_{-L}^L \tilde{\phi} dl. \quad (22)$$

The averaged quasineutrality (current closure) equation (9) becomes

$$-i\omega_0 \bar{n} - i \frac{en_0}{\omega m_i} \frac{\partial^2}{\partial y^2} \bar{\phi} - \frac{1}{eL} J_{0e} \frac{e\bar{\phi}}{T_e} = 0, \quad (23)$$

One mode can be obtained from (23), by using the Boltzmann relation averaged along the magnetic field line tube

$$\bar{n}_e = \frac{e\bar{\phi}}{T_e} n_0. \quad (24)$$

Equations (23) and (24) produce the dispersion equation for the damped mode

$$\omega = \frac{k_y^2 c_s^2}{\omega_0 + i\nu_{sh}}, \quad (25)$$

where $\nu_{sh} \equiv c_s/L$ is the effective "collision frequency" characterizing the sheath resistivity.

However, the conditions (5) and (24) is not the most general solution of the electron momentum balance equation (6). Indeed, integrating (6) along the magnetic field lines one obtains

$$\tilde{n}_e(z, y, t) = \frac{en_0}{T_e} \left(\tilde{\phi}(z, y, t) + \bar{C}(y, t) \right). \quad (26)$$

The integration constant $C(y, t)$, which may depend explicitly on y and t is determined by boundary conditions for perturbations at the sheath. Then the perturbed electron current at the boundary found from (16) is

$$\tilde{J}_e = \frac{e}{T_e} \bar{C} J_{0e}. \quad (27)$$

The density perturbation induced by this current can be found from the electron continuity equation (8) giving

$$-i(\omega - \omega_0)\bar{n}_e + \nu_{sh} \frac{e\bar{C}}{T_e} n_0 = 0. \quad (28)$$

It is important to note that in this process, the plasma potential is also perturbed. This perturbation can be found from (26) and (28):

$$\tilde{\phi} = -\bar{C} \frac{\omega - \omega_0 + i\nu_{sh}}{\omega - \omega_0}. \quad (29)$$

Then, the perturbed plasma density written in terms of the electrostatic potential is

$$\frac{\bar{n}_e}{n_0} = i\nu_{sh} \frac{e}{T_e} \frac{\bar{\phi}}{\omega - \omega_0 + i\nu_{sh}}. \quad (30)$$

We have changed the notation for $\tilde{\phi}$ to $\bar{\phi}$ to emphasize that this perturbation depends only in y and t and is independent of z .

Alternatively, these plasma density perturbations can be found directly from the averaged equation (8) and using (16) for the perturbed electron current

$$-i(\omega - \omega_0)\bar{n}_e - \frac{1}{eL} \left(\frac{\bar{n}}{n_0} - \frac{e\bar{\phi}}{T_e} \right) J_{0e} = 0, \quad (31)$$

which results in the same expression (30).

The perturbed ion density is found from the averaged ion continuity equation

$$-i\omega\bar{n}_i - i\frac{en_0}{\omega m_i} \frac{\partial^2}{\partial y^2} \bar{\phi} + \frac{1}{eL} \frac{\bar{n}}{n_0} J_{0i} = 0, \quad (32)$$

giving

$$\frac{\bar{n}_i}{n_0} = \frac{ek_{\perp}^2 \tilde{\phi}}{\omega(\omega + i\nu_{sh})m_i}. \quad (33)$$

Using quasineutrality, from Eqs. (30) and (33), we obtain the following dispersion relation:

$$\omega^2 + i\omega \left(\nu_{sh} + \frac{k_y^2 c_s^2}{\nu_{sh}} \right) - k_y^2 c_s^2 \left(1 + i\frac{\omega_0}{\nu_{sh}} \right) = 0. \quad (34)$$

In absence of the equilibrium drift ($\omega_0 = 0$), Eq. (34) describe sound waves damped by sheath resistivity

$$\omega = -ik_y^2 c_s^2 / \nu_{sh}. \quad (35)$$

For large $\omega_0 > \nu_{sh}$, it has an unstable root with

$$\omega = \pm (i\omega_0 k_y^2 c_s^2 / \nu_{sh})^{1/2} \quad (36)$$

6. Small scale modes. In this section we consider the sheath instability by taking into account the eigen-mode structure in the direction parallel to the magnetic field. The ion continuity and motion equations result in the following local expression for ion density

$$\tilde{n}_i = \frac{en_0}{\omega^2 m_i} k_{\perp}^2 \tilde{\phi} - \frac{en_0}{\omega^2 m_i} \frac{\partial^2}{\partial z^2} \tilde{\phi}. \quad (37)$$

The parallel component of the electron equation of motion is

$$0 = en_0 \frac{\partial \tilde{\phi}}{\partial z} - T_e \frac{\partial \tilde{n}_e}{\partial z}, \quad (38)$$

which, assuming constant electron temperature gives, and we assume $\tilde{\phi} = \tilde{\phi}(y, t)$

$$\tilde{n}_e = \frac{e\tilde{\phi}}{T_e} n_0. \quad (39)$$

Using quasineutrality and Eqs. (37) and (39), we have

$$\tilde{\phi} = \frac{k_{\perp}^2 c_s^2}{\omega^2} \tilde{\phi} - \frac{c_s^2}{\omega^2} \frac{\partial^2}{\partial z^2} \tilde{\phi} \quad (40)$$

This is an eigen mode equation for ion sound oscillation. Since ions are not subject of $\mathbf{E} \times \mathbf{B}$, there is no Doppler shift in the mode frequency. The eigen mode equation (40) is solved in plane geometry $-L \leq z \leq L$ with boundary conditions at $z = \pm L$. The boundary conditions are defined by the current into the sheath (15) and (16)

$$\tilde{J}_z = \tilde{J}_{zi} + \tilde{J}_{ze} = -\frac{e\tilde{\phi}}{T_e} J_{0e}. \quad (41)$$

The ion and electron current are found from electron and ion equations of motion and continuity equations

$$\tilde{J}_{zi} = -\frac{ie^2 n_0}{m_i \omega} \frac{\partial \tilde{\phi}}{\partial z}, \quad (42)$$

$$-i(\omega - \omega_0) \tilde{n}_e - \frac{1}{e} \frac{\partial}{\partial z} \tilde{J}_{ze} = 0 \quad (43)$$

Boundary conditions (41), (42), and (43) at $z = \pm L$ and (40) make a full eigenmode problem.

General solution for (40) has the form

$$\phi = A \exp(ik_z z) + B \exp(-ik_z z), \quad (44)$$

where k_z satisfy the equation

$$k_z^2 = \frac{\omega^2}{c_s^2} - k_\perp^2. \quad (45)$$

Using (42) and (43) and boundary condition (41) one gets the dispersion relation

$$\frac{k_z c_s}{\omega} \left[1 - \frac{\omega(\omega - \omega_0)}{k_z^2 c_s^2} \right] = \frac{A \exp(ik_z z) + B \exp(-ik_z z)}{A \exp(ik_z z) - B \exp(-ik_z z)} \Big|_{z=\pm L} \quad (46)$$

This dispersion relation can be cast for even $A = B$ and odd $A = -B$ modes in the form

$$\frac{k_z c_s}{\omega} \left[1 - \frac{\omega(\omega - \omega_0)}{k_z^2 c_s^2} \right] = i \tan k_z L, \quad (47)$$

$$\frac{k_z c_s}{\omega} \left[1 - \frac{\omega(\omega - \omega_0)}{k_z^2 c_s^2} \right] = -i \cot k_z L. \quad (48)$$

Consider the long wavelength approximation $k_z L \ll 1$.

$$\frac{k_z c_s}{\omega} \left[1 - \frac{\omega(\omega - \omega_0)}{k_z^2 c_s^2} \right] = \frac{1}{ik_z L} \quad (49)$$

and for odd modes with $A = -B$

$$\frac{k_z c_s}{\omega} \left[1 - \frac{\omega(\omega - \omega_0)}{k_z^2 c_s^2} \right] = ik_z L \quad (50)$$

Using the relation for k_z , the dispersion relation for the even mode in the long wavelength limit becomes

$$(\omega_0 + i\nu_{sh})\omega = k_\perp^2 c_s^2.$$

There is no instability for the even mode in the long wavelength limit.

Odd mode (50) gives the equation for the complex wavenumber k_z

$$k_z^2 c_s^2 = \frac{(\omega - \omega_0)\omega}{1 - i\omega/\nu_{sh}},$$

Using it in the dispersion equation (45) we obtain

$$\omega^3 + (i\nu_{sh}\omega_0 - k_\perp^2 c_s^2)\omega - i\nu_{sh}k_\perp^2 c_s^2 = 0. \quad (51)$$

This has the unstable root instability $\omega = (-i\nu_{sh}\omega_0)^{1/2}$ for $k_\perp \rightarrow 0$. For typical Hall thruster parameters: plasma density $n_0 = 10^{12} \text{ cm}^{-3}$, magnetic field $B_0 = 150 \text{ G}$, electric field $E_0 = 200 \text{ V/cm}$, the electron temperature $T_e = 20 \text{ eV}$, the length scale $L = 1 \text{ cm}$, and lowest wavenumber $k_y = 1.0 \text{ cm}^{-1}$, the unstable mode has the frequency and growth rate of the order of $\omega_r \simeq \gamma \simeq 4 \text{ MHz}$. It scales as $\sim (k_y E_0)^{1/2}$. There is also a weakly unstable root for $\omega_0 \rightarrow 0$ corresponding to

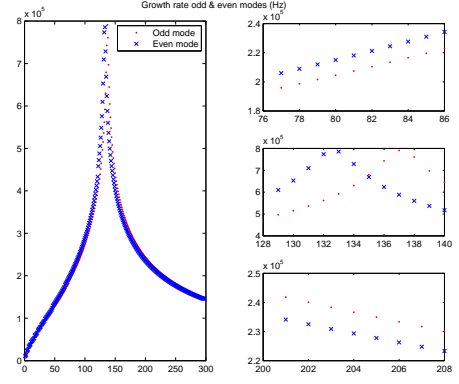


FIG. 1: Growth rate of the unstable modes in Eqs. (47) and (48). The right panel shows zoomed in interspacing of even and odd modes.

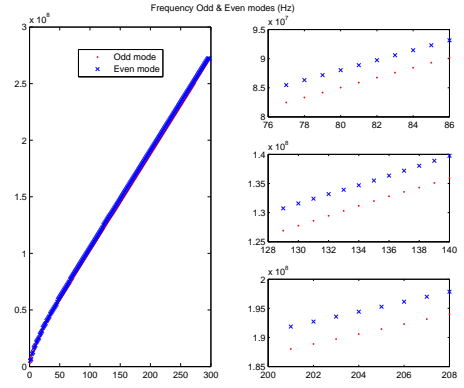


FIG. 2: Real part of the frequency of unstable modes in Eqs. (47) and (48). The right panel shows zoomed in interspacing of even and odd modes.

a weak evolution of the initial equilibrium [15]. The growth rate and real frequency of the unstable mode in 51 are shown in Fig. 1.

The small scale instabilities correspond to the roots of the equations (47) and (48). The numerical solutions are shown in Fig. 2.

7. Summary. To summarize, we have identified novel class of negative energy wave instability operating in Hall plasma with collisionless unmagnetized ions and magnetized electrons in presence of the equilibrium electric field. The instability is triggered by the dissipation due to the normal current into the sheath. Such instabilities may exist in the form of the symmetric global mode, Eq. (51), or small scale modes described by equations (47 and (48). The global mode is slowly varying along the magnetic field, with the characteristic length scale $k_z 1/L$, while small scale modes are characterized by fast variations along the magnetic field lines. The fluctuations

associated with such modes will affect both turbulent mobility of bulk electrons and near-wall conductivity of electrons reflected from the fluctuating sheath.

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